Eur. Phys. J. A **12**, 341–347 (2001)

THE EUROPEAN PHYSICAL JOURNAL A

© Società Italiana di Fisica
Springer-Verlag 2001

# On polarization of strange baryons in reactions $p + p \to p + \Lambda^0 + K^+$ and $p + p \to p + \Sigma^0 + K^+$ near thresholds

A.Ya. Berdnikov, Ya.A. Berdnikov[a], A.N. Ivanov[b,c], V.A. Ivanova[d], V.F. Kosmach[d], M.D. Scadron[e], and N.I. Troitskaya[c]

Atominstitut der Österreichischen Universitäten, Arbeitsbereich Kernphysik und Nukleare Astrophysik, Technische Universität Wien, Wiedner Hauptstr. 8-10, A-1040 Wien, Austria

Received: 22 October 2001
Communicated by V.V. Anisovich

**Abstract.** Polarization properties of strange baryons produced in pp reactions, $p + p \to p + \Lambda^0 + K^+$ and $p + p \to p + \Sigma^0 + K^+$, near thresholds of the final states $p\Lambda^0K^+$ and $p\Sigma^0K^+$ are analysed relative to polarizations of colliding protons. The cross-sections for pp reactions are calculated within the effective Lagrangian approach accounting for strong pp rescattering in the initial state of colliding protons with a dominant contribution of the one-pion exchange and strong final-state interaction of daughter hadrons (Eur. Phys. J. A **9**, 425 (2000)).

**PACS.** 11.10.Cd Axiomatic approach – 25.40.-h Nucleon-induced reactions – 13.88.+e Polarization in interactions and scattering – 14.20.Jn Hyperons

## 1 Introduction

Recently [1] we have considered a production of strangeness in pp reactions, $p + p \to p + Y + K^+$, where $Y = \Lambda^0$ or $\Sigma^0$, near thresholds of daughter hadrons. We have derived the effective Lagrangian

$$\begin{aligned}&\mathcal{L}^{pp\to pYK^+}(x) = \\ &\quad i\,\frac{1}{4}\,C_{pYK^+}\,\varphi^{\dagger}_{K^+}(x)\,\{[\bar{p}(x)\gamma^5 Y^c(x)][\bar{p^c}(x)p(x)] \\ &\quad +[\bar{p}(x)Y^c(x)][\bar{p^c}(x)\gamma^5 p(x)] \\ &\quad +[\bar{p}(x)\gamma^{\mu}Y^c(x)][\bar{p^c}(x)\gamma_{\mu}\gamma^5 p(x)]\}, \qquad (1.1)\end{aligned}$$

describing the effective vertex of the transition $p + p \to Y + K^+ + p$, where $p(x)$, $Y(x)$ and $\varphi_{K^+}(x)$ are the interpolating operators of the proton, hyperon and $K^+$-meson fields, the index (c) stands for a charge conjugation. The first and the last two terms in the Lagrangian equation (1.1) describe the pY-pair coupled in the spin singlet, $S = 0$, and spin triplet state, $S = 1$, respectively. The coupling constant $C_{pYK^+}$ has been calculated in ref. [1] and reads

$$C_{pYK^+} = \frac{g_{pYK^+}g^2_{\pi NN}}{M_p+M_Y+M_{K^+}}\,\frac{1}{M^2_{\pi}+2M_p(E_{\vec{p}}-M_p)}, \qquad (1.2)$$

where $E_{\vec{p}} = \sqrt{\vec{p}^{\,2} + M^2_p}\,|_{\vec{p}\,=\,\vec{p}_0}$ and $p_0 = \sqrt{(M_Y + M_{K^+} - M_p)(M_Y + M_{K^+} + 3M_p)}/2$ is the relative 3-momentum of the colliding protons near threshold, $g_{pYK^+}$ and $g_{\pi NN}$ are the *pseudoscalar* meson-baryon-baryon coupling constants [1,2]. Then, $M_p$, $M_Y$ and $M_{K^+}$ are masses of the proton, the hyperon and the $K^+$-meson. The appearance of the $\pi$-meson mass $M_{\pi}$ testifies the calculation of the effective coupling constant $C_{pYK^+}$ in the one-pion exchange approximation. As has been shown in ref. [1] the accuracy of this approximation makes up a few percent.

According to relativistically covariant partial-wave analysis developed by Anisovich *et al.* [3] the spin triplet state, $S = 1$, of the pY-pair is splitted into the $^3P_0$ state, described by the the second term in (1.1), and the $^3S_1$ and $^3D_1$ states mixed in the third term of (1.1).

[a] e-mail: berdnikov@twonet.stu.neva.ru. *Present address*: Department of Nuclear Physics, State Technical University, 195251 St. Petersburg, Russian Federation.

[b] e-mail: ivanov@kph.tuwien.ac.at.

[c] *Permanent address*: Department of Nuclear Physics, State Technical University, 195251 St. Petersburg, Russian Federation.

[d] *Present address*: Department of Nuclear Physics, State Technical University, 195251 St. Petersburg, Russian Federation.

[e] e-mail: scadron@physics.arizona.edu. *Present address*: Physics Department, University of Arizona, Tucson, AZ 85721, USA.

In ref. [1] the cross-sections for the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$, calculated for unpolarized particles, fit experimental data [3–6] with accuracy better than 11% for excess of energy $\varepsilon$, defined by $\varepsilon=\sqrt{s}-M_{\mathrm{p}}-M_{\mathrm{Y}}-M_{\mathrm{K}^+}$ [1], ranging values from the region $0.68\,\mathrm{MeV}\le\varepsilon\le 138\,\mathrm{MeV}$ [1].

In this paper, we calculate the cross-sections for the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$, where $\mathrm{Y}=\Lambda^0$ or $\Sigma^0$, near thresholds in dependence on polarizations of baryons. We analyse the contributions of the pY-pair produced in the ${}^1S_0$, ${}^3P_0$, ${}^3S_1$ and ${}^3D_1$ states. We show that the pY-pair can be created only in the spin singlet ${}^1S_0$ and spin triplet ${}^3S_1$ states. Therewith, a production of a polarized strange baryon relative to polarizations of colliding protons comes about only for the spin triplet ${}^3S_1$ state of the pY-pair.

The paper is organized as follows. In sect. 2 the projection operators introduced by Anisovich *et al.* [3] for the projection of the wave function of a nucleon-nucleon pair onto the ${}^3S_1$ and ${}^3D_1$ states are generalized for the case of non-equal masses of coupled baryons. In sect. 3 we calculate the amplitude of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$. We show that near threshold the pY-pair can be produced only in the spin singlet ${}^1S_0$ and spin triplet ${}^3S_1$ states. This corresponds the colliding protons coupled in the ${}^3P_0$ and ${}^3P_1$ states, respectively. In sect. 4 we calculate the cross-sections for pp reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda^0+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$ in dependence on polarizations of colliding protons and strange baryons $\Lambda^0$ and $\Sigma^0$. In the conclusion we discuss the obtained results.

## 2 Partial-wave decomposition of the effective vertex of the transition $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$

The calculation of the amplitude of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$, we start with the decomposition of the effective vertex of the transition $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$, described by the effective Lagrangian (1.1), into the interactions for which the pY-pair couples to the initial protons and the $\mathrm{K}^+$-meson in the states with certain orbital momenta. For this aim it is convenient to pass into momentum representation. In momentum representation the effective vertex described by the effective Lagrangian (1.1) reads [1]

$$\begin{aligned}\mathcal{M}(\mathrm{pp}\to\mathrm{pYK}^+) &= i\,\frac{1}{2}\,C_{\mathrm{pYK}^+}\\ &\times\Big\{\Big[\bar u\Big(-\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{p}}\Big)\gamma^5 u^c\Big(\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{Y}}\Big)\Big]\\ &\times[\bar{u^c}(-\vec p,\alpha_2)u(\vec p,\alpha_1)]\\ &+\Big[\bar u\Big(-\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{p}}\Big)u^c\Big(\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{Y}}\Big)\Big]\\ &\times[\bar{u^c}(-\vec p,\alpha_2)\gamma^5 u(\vec p,\alpha_1)]\\ &+\Big[\bar u\Big(-\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{p}}\Big)\gamma^\mu u^c\Big(\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{Y}}\Big)\Big]\\ &\times[\bar{u^c}(-\vec p,\alpha_2)\gamma_\mu\gamma^5 u(\vec p,\alpha_1)]\Big\}\,, \end{aligned}\qquad (2.1)$$

According to classification given by Anisovich *et al.* [3] the first, second and third terms in the r.h.s. of (2.1) describe the contribution of the pY-pair coupled in the ${}^1S_0$, ${}^3P_0$ and a mixture of ${}^3S_1$ and ${}^3P_1$ states, respectively. In the low-energy limit there survive only the first and the third terms of (2.1). Indeed, the wave function of the pY-pair in the ${}^3P_0$ state is proportional to a relative 3-momentum of the pY-pair and vanishes in the low-energy limit. Therefore, near threshold of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$, the second term in (2.1) can be dropped out. This gives

$$\begin{aligned}\mathcal{M}(\mathrm{pp}\to\mathrm{pYK}^+) &= i\,\frac{1}{2}\,C_{\mathrm{pYK}^+}\\ &\times\Big\{\Big[\bar u\Big(-\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{p}}\Big)\gamma^5 u^c\Big(\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{Y}}\Big)\Big]\\ &\times\Big[\bar{u^c}(-\vec p,\alpha_2)u(\vec p,\alpha_1)\Big]\\ &+\Big[\bar u\Big(-\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{p}}\Big)\gamma^\mu u^c\Big(\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}},\alpha_{\mathrm{Y}}\Big)\Big]\\ &\times\Big[\bar{u^c}(-\vec p,\alpha_2)\gamma_\mu\gamma^5 u(\vec p,\alpha_1)\Big]\Big\}\,, \end{aligned}\qquad (2.2)$$

For the decomposition of the wave function of the pY-pair in the last term of (2.2) into the states ${}^3S_1$ and ${}^3D_1$ with certain orbital momenta we introduce the notations: $k_{\mathrm{Y}}=(E_{\mathrm{Y}},\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}})=(E_{\mathrm{Y}},\vec k_{\mathrm{Y}})$, $k_{\mathrm{p}}=(E_{\mathrm{p}},-\vec q_{\mathrm{pY}}-\frac12\,\vec p_{\mathrm{K}})=(E_{\mathrm{p}},\vec k_{\mathrm{p}})$, $P=k_{\mathrm{Y}}+k_{\mathrm{p}}$, $k=\frac12\,(k_{\mathrm{Y}}-k_{\mathrm{p}})$ and

$$\gamma^\perp_\mu=\gamma_\mu-\hat P\,\frac{P_\mu}{P^2}\,,\qquad k^\perp_\mu=k_\mu-\frac{P\cdot k}{P^2}\,P_\mu. \qquad (2.3)$$

The 4-vectors $\gamma^\perp_\mu$ and $k^\perp_\mu$ are orthogonal to $P_\mu$: $P\cdot\gamma^\perp=P\cdot k^\perp=0$.

The baryon densities describing the pY-pair in the ${}^3S_1$ and ${}^3D_1$ states are defined by [3] (see also [7])

$$\begin{aligned}\Psi_\mu({}^3S_1;\alpha_{\mathrm{p}},\alpha_{\mathrm{Y}})&=[\bar u(k_{\mathrm{p}},\alpha_{\mathrm{p}})S_\mu u^c(k_{\mathrm{Y}},\alpha_{\mathrm{Y}})],\\ \Psi_\mu({}^3D_1;\alpha_{\mathrm{p}},\alpha_{\mathrm{Y}})&=[\bar u(k_{\mathrm{p}},\alpha_{\mathrm{p}})D_\mu u^c(k_{\mathrm{Y}},\alpha_{\mathrm{Y}})],\end{aligned}\qquad (2.4)$$

where $S_\mu$ and $D_\mu$ are relativistically covariant operators of the projection onto the ${}^3S_1$ and ${}^3D_1$ states, respectively:

$$\begin{aligned}S_\mu&=\frac{1}{\sqrt2}\,\frac{1}{\sqrt{P^2-(M_{\mathrm{Y}}-M_{\mathrm{p}})^2}}\\ &\quad\times\Big[\gamma^\perp_\mu+\frac{2}{M_{\mathrm{Y}}+M_{\mathrm{p}}+\sqrt{P^2}}\,k^\perp_\mu\Big],\\ D_\mu&=\frac{2}{\sqrt{P^2-(M_{\mathrm{Y}}-M_{\mathrm{p}})^2}}\Big[\frac14\Big(1-\frac{(M_{\mathrm{Y}}+M_{\mathrm{p}})^2}{P^2}\Big)\\ &\quad\times\Big(1-\frac{(M_{\mathrm{Y}}-M_{\mathrm{p}})^2}{P^2}\Big)\gamma^\perp_\mu-\frac{1}{P^2}\Big(1-\frac{(M_{\mathrm{Y}}-M_{\mathrm{p}})^2}{P^2}\Big)\\ &\quad\times\Big(\sqrt{P^2}+\frac{M_{\mathrm{Y}}+M_{\mathrm{p}}}{2}\Big)\,k^\perp_\mu\Big].\end{aligned}\qquad (2.5)$$

This is the generalization of the projection operators introduced by Anisovich *et al.* (see (C.2-C.3) of ref. [3]) for non-equal masses of coupled baryons. In the center-of-mass frame of the pY-pair the baryon densities (2.4)

$$\frac{1}{3}\int \mathrm{tr}\{L_\mu(\hat{k}_\mathrm{p}+M_\mathrm{p})L^\mu(-\hat{k}_\mathrm{Y}+M_\mathrm{Y})\}\,(2\pi)^4\delta^{(4)}(P-k_\mathrm{p}-k_\mathrm{Y})\,\frac{\mathrm{d}^3k_\mathrm{p}}{(2\pi)^3 2E_\mathrm{p}}\,\frac{\mathrm{d}^3k_\mathrm{Y}}{(2\pi)^3 2E_\mathrm{Y}} = \begin{cases} \dfrac{1}{8\pi}\left(1-\dfrac{(M_\mathrm{Y}-M_\mathrm{p})^2}{P^2}\right)^{1/2}\left(1-\dfrac{(M_\mathrm{Y}+M_\mathrm{p})^2}{P^2}\right)^{1/2}, & L_\mu = S_\mu, \\ \dfrac{1}{8\pi}\left(1-\dfrac{(M_\mathrm{Y}-M_\mathrm{p})^2}{P^2}\right)^{5/2}\left(1-\dfrac{(M_\mathrm{Y}+M_\mathrm{p})^2}{P^2}\right)^{5/2}, & L_\mu = D_\mu. \end{cases} \quad (2.9)$$

are equal to

$$\Psi_0(^3S_1;\alpha_\mathrm{p},\alpha_\mathrm{Y}) = [\bar{u}(k_\mathrm{p},\alpha_\mathrm{p})S_0 u^\mathrm{c}(k_\mathrm{Y},\alpha_\mathrm{Y})] = 0,$$
$$\vec{\Psi}(^3S_1;\alpha_\mathrm{p},\alpha_\mathrm{Y}) = [\bar{u}(k_\mathrm{p},\alpha_\mathrm{p})\vec{S}u^\mathrm{c}(k_\mathrm{Y},\alpha_\mathrm{Y})] = \frac{1}{\sqrt{2}}\varphi^\dagger_\mathrm{p}(\alpha_\mathrm{p})\vec{\sigma}\varphi_\mathrm{Y}(\alpha_\mathrm{Y}),$$
$$\Psi_0(^3D_1;\alpha_\mathrm{p},\alpha_\mathrm{Y}) = [\bar{u}(k_\mathrm{p},\alpha_\mathrm{p})D_0 u^\mathrm{c}(k_\mathrm{Y},\alpha_\mathrm{Y})] = 0,$$
$$\vec{\Psi}(^3D_1;\alpha_\mathrm{p},\alpha_\mathrm{Y}) = [\bar{u}(k_\mathrm{p},\alpha_\mathrm{p})\vec{D}u^\mathrm{c}(k_\mathrm{Y},\alpha_\mathrm{Y})] = -v^2\varphi^\dagger_\mathrm{p}(\alpha_\mathrm{p})\left(\frac{3(\vec{\sigma}\cdot\vec{n})\vec{n}-\vec{\sigma}}{2}\right)\varphi_\mathrm{Y}(\alpha_\mathrm{Y}), \quad (2.6)$$

where $\vec{n} = \vec{p}/|\vec{p}\,|$ is a unit vector of a relative momentum $\vec{p}$ and $v$ amounts to

$$v = \sqrt{\left(1-\frac{(M_\mathrm{Y}-M_\mathrm{p})^2}{P^2}\right)\left(1-\frac{(M_\mathrm{Y}+M_\mathrm{p})^2}{P^2}\right)}. \quad (2.7)$$

Hence, the formulas (2.6) demonstrate that the baryon densities (2.4) describe the pY-pair in the $S$- and $D$-wave states with a total spin $S = 1$ and a total momentum $J = 1$. The baryon densities (2.6) are normalized by [7]

$$\frac{1}{3}\sum_{\alpha_\mathrm{p}=\pm 1/2}\ \sum_{\alpha_\mathrm{Y}=\pm 1/2} \vec{\Psi}^\dagger(^3S_1;\alpha_\mathrm{p},\alpha_\mathrm{Y})\cdot\vec{\Psi}(^3S_1;\alpha_\mathrm{p},\alpha_\mathrm{Y}) = 1,$$
$$\frac{1}{3}\sum_{\alpha_\mathrm{p}=\pm 1/2}\ \sum_{\alpha_\mathrm{Y}=\pm 1/2} \vec{\Psi}^\dagger(^3D_1;\alpha_\mathrm{p},\alpha_\mathrm{Y})\cdot\vec{\Psi}(^3D_1;\alpha_\mathrm{p},\alpha_\mathrm{Y}) = v^4, \quad (2.8)$$

where $v$ is given by (2.7). The factor 3 in the denominator of the l.h.s. of eq. (2.8) describes the number of the states of the pY-pair with a total momentum $J = 1$, $2J+1 = 3$.

For the analysis of nuclear reactions it is convenient to remind that the normalization (2.8) corresponds to the normalization in the phase volume of the pY-pair [3] (see also [7]):

*see eq.* (2.9) *above.*

Solving equations (2.5) with respect to $\gamma^\perp_\mu$ we express $\gamma^\perp_\mu$ in terms of the projection operators $S_\mu$ and $D_\mu$

$$\gamma^\perp_\mu = \frac{2\sqrt{2}}{3}\left(\sqrt{P^2}+\frac{M_\mathrm{Y}+M_\mathrm{p}}{2}\right)\sqrt{1-\frac{(M_\mathrm{Y}-M_\mathrm{p})^2}{P^2}}\,S_\mu + \frac{2}{3}\frac{(P^2)^{3/2}}{(\sqrt{P^2}+M_\mathrm{Y}+M_\mathrm{p})\sqrt{P^2-(M_\mathrm{Y}-M_\mathrm{p})^2}}\,D_\mu. \quad (2.10)$$

In the limit of equal masses $M_\mathrm{Y} = M_\mathrm{p} = M_\mathrm{N}$ the r.h.s. of (2.10) reduces itself to the form of eq. (2.16) of ref. [7].

Near threshold of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ we can define $\sqrt{P^2}$ in terms of an excess of energy $\varepsilon$: $\sqrt{P^2} = \varepsilon + M_\mathrm{Y} + M_\mathrm{p}$. Hence, near threshold of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ the Dirac matrix $\gamma^\perp_\mu$ expanded into the projection operators $S_\mu$ and $D_\mu$ can be approximated by

$$\gamma^\perp_\mu = 2\sqrt{2}\,\sqrt{M_\mathrm{Y}M_\mathrm{p}}\,S_\mu + \frac{(M_\mathrm{Y}+M_\mathrm{p})^2}{6\sqrt{M_\mathrm{Y}M_\mathrm{p}}}\,D_\mu. \quad (2.11)$$

Since in the low-energy limit the parameter $v$ is of order $O(\varepsilon)$, $v \sim \varepsilon/(M_\mathrm{Y}+M_\mathrm{p})$, the contribution of the $^3D_1$ states can be neglected near threshold of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$.

Substituting (2.11) in (2.2) and keeping only leading terms in the low-energy limit we arrive at the effective vertex of the transition $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ given by

$$\mathcal{M}(\mathrm{pp}\to\mathrm{pYK}^+) = i\,\frac{1}{2}\,C_{\mathrm{pYK}^+}\times\Big\{[\bar{u}(\vec{k}_\mathrm{p},\alpha_\mathrm{p})\gamma^5 u^\mathrm{c}(\vec{k}_\mathrm{Y},\alpha_\mathrm{Y})][\bar{u^\mathrm{c}}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)] - 2\sqrt{2}\,\sqrt{M_\mathrm{Y}M_\mathrm{p}}[\bar{u}(\vec{k}_\mathrm{p},\alpha_\mathrm{p})\,\vec{S}\,u^\mathrm{c}(\vec{k}_\mathrm{Y},\alpha_\mathrm{Y})]\cdot[\bar{u^\mathrm{c}}(-\vec{p},\alpha_2)\vec{\gamma}\gamma^5 u(\vec{p},\alpha_1)]\Big\}. \quad (2.12)$$

We have taken into account the fact that near threshold, when we are able to neglect a 3-momentum of the $\mathrm{K}^+$-meson, the pY-pair is practically in the center-of-mass frame. This implies that only spatial components of the projection operator $S_\mu$ are material.

The effective vertex (2.12) evidences that near threshold of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ the pY-pair can be produced only in the spin singlet $^1S_0$ and spin triplet $^3S_1$ states. This corresponds to colliding protons coupled in the $^3P_0$ and $^3P_1$ states, respectively.

## 3 Amplitude of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$

For the calculation of the amplitude of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ we would follow ref. [1] and take into account strong pp interaction in the initial state, *i.e.* pp rescattering $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{p}$. As has been shown in ref. [1]

the effective pp interaction responsible for the transition $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{p}$ can be represented in the local form

$$\begin{aligned}\mathcal{L}^{\mathrm{pp}\to\mathrm{pp}}(x) = \frac{1}{8}\,C_{\mathrm{pp}}\Big\{&[\bar{p}(x)p^{\mathrm{c}}(x)][\bar{p^{\mathrm{c}}}(x)p(x)]\\ &+[\bar{p}(x)\gamma^5 p^{\mathrm{c}}(x)][\bar{p^{\mathrm{c}}}(x)\gamma^5 p(x)]\\ &+[\bar{p}(x)\gamma_\mu\gamma^5 p^{\mathrm{c}}(x)][\bar{p^{\mathrm{c}}}(x)\gamma^\mu\gamma^5 p(x)]\Big\},\end{aligned} \tag{3.1}$$

where the coupling constant $C_{\mathrm{pp}}$ is equal to [1]

$$C_{\mathrm{pp}} = \frac{g^2_{\pi\mathrm{NN}}}{4\vec{p}^{\,2}}\ln\left(1+\frac{4\vec{p}^{\,2}}{M^2_\pi}\right). \tag{3.2}$$

By summing up infinite series of one-proton loop diagrams the vertices of which are defined by the effective interaction eq. (3.1) we arrive at the expressions [1,8]

$$\begin{aligned}&[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)]\to\\ &\qquad\frac{[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)]}{1+\dfrac{C_{\mathrm{pp}}}{64\pi^2}\displaystyle\int\frac{\mathrm{d}^4k}{\pi^2 i}\,\mathrm{tr}\left\{\frac{1}{M_{\mathrm{p}}-\hat{k}}\,\frac{1}{M_{\mathrm{p}}-\hat{k}-\hat{Q}}\right\}},\\ &[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)\gamma^i\gamma^5 u(\vec{p},\alpha_1)]\to\\ &\qquad(D^{-1}_{\mathrm{pp}}(Q))^{ij}[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2))\gamma_j\gamma^5 u(\vec{p},\alpha_1)],\\ &D^{ij}_{\mathrm{pp}}(Q) = g^{ji}+\frac{C_{\mathrm{pp}}}{64\pi^2}\\ &\qquad\times\int\frac{\mathrm{d}^4k}{\pi^2 i}\,\mathrm{tr}\left\{\gamma^i\gamma^5\frac{1}{M_{\mathrm{p}}-\hat{k}}\gamma^j\gamma^5\frac{1}{M_{\mathrm{p}}-\hat{k}-\hat{Q}}\right\},\end{aligned} \tag{3.3}$$

where Latin indices run over $i = 1,2,3$ and $Q = \left(2\sqrt{\vec{p}^{\,2}+M^2_{\mathrm{p}}},\vec{0}\right)$.

After the evaluation of momentum integrals and renormalization of wave functions of the protons [1, 8], we obtain the contributions of strong pp interaction, pp rescattering, in the initial state of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$:

$$\begin{aligned}&[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)]\to\\ &\qquad\frac{[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)]}{1+\dfrac{C_{\mathrm{pp}}(\vec{p}^{\,2},\Lambda)}{8\pi^2}\,\dfrac{|\vec{p}\,|^3}{E_{\vec{\mathrm{p}}}}\left[\ln\left(\dfrac{E_{\vec{\mathrm{p}}}+|\vec{p}\,|}{E_{\vec{\mathrm{p}}}-|\vec{p}\,|}\right)+\pi\,i\right]} =\\ &[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)]\,f^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_0;|\vec{p}\,|)\,e^{i\delta^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_0;|\vec{p}\,|)},\\ &[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)\vec{\gamma}\,\gamma^5 u(\vec{p},\alpha_1)]\to\\ &\qquad\frac{[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2))\vec{\gamma}\,\gamma^5 u(\vec{p},\alpha_1)]}{1+\dfrac{C_{\mathrm{pp}}(\vec{p}^{\,2},\Lambda)}{8\pi^2}\,\dfrac{|\vec{p}\,|^3}{E_{\vec{\mathrm{p}}}}\left[\ln\left(\dfrac{E_{\vec{\mathrm{p}}}+|\vec{p}\,|}{E_{\vec{\mathrm{p}}}-|\vec{p}\,|}\right)+\pi\,i\right]} =\\ &\qquad[\bar{u^{\mathrm{c}}}(-\vec{p},\alpha_2)\vec{\gamma}\,\gamma^5 u(\vec{p},\alpha_1)]\\ &\qquad\times f^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_1;|\vec{p}\,|)\,e^{i\delta^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_1;|\vec{p}\,|)},\end{aligned} \tag{3.4}$$

where $C_{\mathrm{pp}}(\vec{p}^{\,2},\Lambda)$ amounts to [1]

$$C_{\mathrm{pp}}(\vec{p}^{\,2},\Lambda) = \frac{C_{\mathrm{pp}}}{1+\dfrac{C_{\mathrm{pp}}\vec{p}^{\,2}}{4\pi^2}\left[\ln\left(\dfrac{\Lambda}{M_{\mathrm{p}}}+\sqrt{1+\dfrac{\Lambda^2}{M^2_{\mathrm{p}}}}\right)-\dfrac{\Lambda}{\sqrt{M^2_{\mathrm{p}}+\Lambda^2}}\right]}. \tag{3.5}$$

The appearance of the cut-off $\Lambda$ is caused by non-trivial $\vec{p}$-dependent logarithmically divergent contributions. The cut-off $\Lambda$ restricts from above 3-momenta of virtual proton fluctuations and is equal to $\Lambda = 1200\,\mathrm{MeV}$ [1].

In our model the amplitudes of pp rescattering in the ${}^3P_0$ and ${}^3P_1$ states are equal near threshold of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$. Therefore, below we denote

$$\begin{aligned}&f^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_0;|\vec{p}\,|)\,e^{i\delta^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_0;|\vec{p}\,|)} =\\ &f^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_1;|\vec{p}\,|)\,e^{i\delta^{\mathrm{pYK}^+}_{\mathrm{pp}}({}^3P_1;|\vec{p}\,|)} =\\ &f^{\mathrm{pYK}^+}_{\mathrm{pp}}(|\vec{p}\,|)\,e^{i\delta^{\mathrm{pYK}^+}_{\mathrm{pp}}(|\vec{p}\,|)} =\\ &\frac{1}{1+\dfrac{C_{\mathrm{pp}}(\vec{p}^{\,2},\Lambda)}{8\pi^2}\,\dfrac{|\vec{p}\,|^3}{E_{\vec{\mathrm{p}}}}\left[\ln\left(\dfrac{E_{\vec{\mathrm{p}}}+|\vec{p}\,|}{E_{\vec{\mathrm{p}}}-|\vec{p}\,|}\right)+\pi\,i\right]}.\end{aligned} \tag{3.6}$$

As has been shown in ref. [1] in our model the amplitude of strong low-energy pY interaction in the final state can be represented in Watson's form for the final-state interaction [9] in terms of the scattering length $a_{\mathrm{pY}}$ and the effective range $r_{\mathrm{pY}}$ of low-energy elastic pY scattering:

$$\begin{aligned}f^{\mathrm{pY}\to\mathrm{pY}}(q_{\mathrm{pY}}) &= \frac{1}{1-\dfrac{1}{2}\,a_{\mathrm{pY}}r_{\mathrm{pY}}q^2_{\mathrm{pY}}+i\,a_{\mathrm{pY}}q_{\mathrm{pY}}} =\\ &\qquad f_{\mathrm{pY}}(q_{\mathrm{pY}})\,e^{i\delta_{\mathrm{pY}}(q_{\mathrm{pY}})}.\end{aligned} \tag{3.7}$$

According to Balewski *et al.* [9] for the description of the final pY interaction in the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ we would use average values for scattering lengths and effective ranges in the spin singlet ${}^1S_0$ and spin triplet ${}^3S_1$ states of the pY-pair: $a_{\mathrm{pY}} = -2.0\,\mathrm{fm}$ and $r_{\mathrm{pY}} = 1.0\,\mathrm{fm}$ [9]. This assumes that the amplitudes of low-energy elastic pY scattering in the spin singlet ${}^1S_0$ and spin triplet ${}^3S_1$ states are equal

$$\begin{aligned}&f_{\mathrm{pY}}({}^1S_0;q_{\mathrm{pY}})\,e^{i\delta_{\mathrm{pY}}({}^1S_0;q_{\mathrm{pY}})} =\\ &f_{\mathrm{pY}}({}^3S_1;q_{\mathrm{pY}})\,e^{i\delta_{\mathrm{pY}}({}^3S_1;q_{\mathrm{pY}})} =\\ &f_{\mathrm{pY}}(q_{\mathrm{pY}})\,e^{i\delta_{\mathrm{pY}}(q_{\mathrm{pY}})}.\end{aligned} \tag{3.8}$$

In ref. [1] we have shown that this assumption agrees well with experimental data [4–6,10].

Accounting for the Coulomb repulsion between the daughter proton and the $\mathrm{K}^+$-meson we obtain the total amplitude of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ near threshold of the final state

$$\mathcal{M}(\mathrm{pp}\to\mathrm{pYK^+}) = \frac{i}{2}\,C_{\mathrm{pYK^+}} \times f^{\mathrm{pYK^+}}_{\mathrm{pp}}(|\vec{p}\,|)\, f_{\mathrm{pY}}(q_{\mathrm{pY}})\, e^{i\delta^{\mathrm{pYK^+}}_{\mathrm{pp}}(|\vec{p}\,|) + i\delta_{\mathrm{pY}}(q_{\mathrm{pY}})} \times \sqrt{\frac{M_{\mathrm{pK^+}}}{q_{\mathrm{pK^+}}}\,\frac{2\pi\alpha}{e^{2\pi\alpha M_{\mathrm{pK^+}}/q_{\mathrm{pK^+}}}-1}} \times\{[\bar{u}(\vec{k}_{\mathrm{p}},\alpha_{\mathrm{p}})\gamma^5 u^c(\vec{k}_{\mathrm{Y}},\alpha_{\mathrm{Y}})][\bar{u^c}(-\vec{p},\alpha_2)u(\vec{p},\alpha_1)] -2\sqrt{2}\,\sqrt{M_{\mathrm{Y}}M_{\mathrm{p}}}[\bar{u}(\vec{k}_{\mathrm{p}},\alpha_{\mathrm{p}}\Big)\,\vec{S}\,u^c(\vec{k}_{\mathrm{Y}},\alpha_{\mathrm{Y}})] \cdot[\bar{u^c}(-\vec{p},\alpha_2)\vec{\gamma}\,\gamma^5 u(\vec{p},\alpha_1)]\}, \quad (3.9)$$

where the factor depending of the fine structure constant $\alpha = 1/137$ takes into account the Coulomb repulsion between the daughter proton and the $\mathrm{K^+}$-meson at low relative 3-momenta $q_{\mathrm{pK^+}}$ [9] (see also [8]), $M_{\mathrm{pK^+}} = M_{\mathrm{p}}M_{\mathrm{K^+}}/(M_{\mathrm{p}}+M_{\mathrm{K^+}})$ is a reduced mass of the $\mathrm{pK^+}$ system.

## 4 Cross-sections for the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda^0+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$ with polarized baryons

The calculation of the cross-section for the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{Y}+\mathrm{K}^+$ we carry out in dependence on polarizations of strange baryon and colliding protons [11]. The polarization 4-vectors of coupled baryons, we define as follows [12]:

$$\zeta^{\mu}_1 = \Big(+\frac{\vec{p}\cdot\vec{\zeta}_1}{M_{\mathrm{p}}}, \vec{\zeta}_1 + \frac{\vec{p}(\vec{p}\cdot\vec{\zeta}_1)}{M_{\mathrm{p}}(E_{\vec{\mathrm{p}}}+M_{\mathrm{p}})}\Big),$$
$$\zeta^{\mu}_2 = \Big(-\frac{\vec{p}\cdot\vec{\zeta}_2}{M_{\mathrm{p}}}, \vec{\zeta}_2 + \frac{\vec{p}(\vec{p}\cdot\vec{\zeta}_2)}{M_{\mathrm{p}}(E_{\vec{\mathrm{p}}}+M_{\mathrm{p}})}\Big),$$
$$\zeta^{\mu}_{\mathrm{Y}} = (0,\vec{\zeta}_{\mathrm{Y}}), \quad (4.1)$$

where $\vec{\zeta}_i\,(i=1,2,\mathrm{Y})$ are polarization 3-vectors of baryons normalized to unity $\vec{\zeta}^{\,2}_i = 1$.

Introducing the polarization 4-vectors of baryons in a standard way

$$\sum_{\alpha_1=\pm1/2} u(p_1,\alpha_1)\bar{u}(p_1,\alpha_1) = (\hat{p}_1+M_{\mathrm{p}})\Big(\frac{1+\gamma^5\hat{\zeta}_1}{2}\Big),$$
$$\sum_{\alpha_2=\pm1/2} u^c(p_2,\alpha_2)\bar{u^c}(p_2,\alpha_2) = (\hat{p}_2-M_{\mathrm{p}})\Big(\frac{1+\gamma^5\hat{\zeta}_2}{2}\Big),$$
$$\sum_{\alpha_{\mathrm{Y}}=\pm1/2} u^c(k_{\mathrm{Y}},\alpha_{\mathrm{Y}})\bar{u^c}(k_{\mathrm{Y}},\alpha_{\mathrm{Y}}) = (\hat{k}_{\mathrm{Y}}-M_{\mathrm{Y}})\Big(\frac{1+\gamma^5\hat{\zeta}_{\mathrm{Y}}}{2}\Big), \quad (4.2)$$

we calculate the squared amplitude (3.9), averaged and summed over the states of colliding protons and final baryons. The result reads

$$\overline{|\mathcal{M}(\mathrm{pp}\to\mathrm{pYK^+})|^2} = C^2_{\mathrm{p}\Lambda\mathrm{K^+}}\,|f^{\mathrm{pYK^+}}_{\mathrm{pp}}(|\vec{p}\,|)|^2|f_{\mathrm{pY}}(q_{\mathrm{pY}})|^2 \times\frac{M_{\mathrm{pK^+}}}{q_{\mathrm{pK^+}}}\,\frac{2\pi\alpha}{e^{2\pi\alpha M_{\mathrm{pK^+}}/q_{\mathrm{pK^+}}}-1} \times 4\vec{p}^{\,2}M_{\mathrm{p}}M_{\mathrm{Y}}\Big(1+\frac{1}{3}\,\vec{\zeta}_1\cdot\vec{\zeta}_2+\frac{1}{3}\,\vec{n}\cdot(\vec{\zeta}_1+\vec{\zeta}_2)(\vec{n}\cdot\vec{\zeta}_{\mathrm{Y}})\Big) = \times\overline{|\mathcal{M}(\mathrm{pp}\to\mathrm{pYK^+})|^2}_0 \times\Big(1+\frac{1}{3}\,\vec{\zeta}_1\cdot\vec{\zeta}_2+\frac{1}{3}\,\vec{n}\cdot(\vec{\zeta}_1+\vec{\zeta}_2)(\vec{n}\cdot\vec{\zeta}_{\mathrm{Y}})\Big), \quad (4.3)$$

where $\overline{|\mathcal{M}(\mathrm{pp}\to\mathrm{pYK^+})|^2}_0$ is a squared amplitude of the reaction under consideration with unpolarized particles and $\vec{n} = \vec{p}/|\vec{p}\,|$ is a unit vector along a relative 3-momentum of colliding protons.

If only one of the colliding protons is polarized, the amplitude (4.3) reduces to a simpler form

$$\overline{|\mathcal{M}(\mathrm{pp}\to\mathrm{pYK^+})|^2} = \overline{|\mathcal{M}(\mathrm{pp}\to\mathrm{pYK^+})|^2}_0 \times\Big(1+\frac{1}{3}\,(\vec{n}\cdot\vec{\zeta})(\vec{n}\cdot\vec{\zeta}_{\mathrm{Y}})\Big), \quad (4.4)$$

where $\vec{\zeta}$ is a polarization vector of the polarized proton in the initial state.

Using (4.3), (4.4) and the results obtained in ref. [1] we write down the cross-sections for the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda^0+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$, when i) colliding protons and a strange baryon are polarized, $\vec{\mathrm{p}}+\vec{\mathrm{p}}\to\mathrm{p}+\vec{\mathrm{Y}}+\mathrm{K}^+$, and ii) there are polarized only one of the colliding protons and a strange baryon, $\vec{\mathrm{p}}+\mathrm{p}\to\mathrm{p}+\vec{\mathrm{Y}}+\mathrm{K}^+$ or $\mathrm{p}+\vec{\mathrm{p}}\to\mathrm{p}+\vec{\mathrm{Y}}+\mathrm{K}^+$:

$$\sigma^{\vec{\mathrm{p}}\,\vec{\mathrm{p}}\to\mathrm{p}\vec{\Lambda}^0\mathrm{K^+}}(\varepsilon) = \sigma^{\mathrm{pp}\to\mathrm{p}\Lambda^0\mathrm{K^+}}(\varepsilon) \times\Big(1+\frac{1}{3}\,\vec{\zeta}_1\cdot\vec{\zeta}_2+\frac{1}{3}\,\vec{n}\cdot(\vec{\zeta}_1+\vec{\zeta}_2)(\vec{n}\cdot\vec{\zeta}_{\Lambda^0})\Big),$$
$$\sigma^{\vec{\mathrm{p}}\,\vec{\mathrm{p}}\to\mathrm{p}\vec{\Sigma}^0\mathrm{K^+}}(\varepsilon) = \sigma^{\mathrm{pp}\to\mathrm{p}\Sigma^0\mathrm{K^+}}(\varepsilon) \times\Big(1+\frac{1}{3}\,\vec{\zeta}_1\cdot\vec{\zeta}_2+\frac{1}{3}\,\vec{n}\cdot(\vec{\zeta}_1+\vec{\zeta}_2)(\vec{n}\cdot\vec{\zeta}_{\Sigma^0})\Big). \quad (4.5)$$

For reactions $\mathrm{p}+\vec{\mathrm{p}}\to\mathrm{p}+\vec{\Lambda}^0+\mathrm{K}^+$ and $\mathrm{p}+\vec{\mathrm{p}}\to\mathrm{p}+\vec{\Sigma}^0+\mathrm{K}^+$ with one polarized proton in the initial state and a polarized strange baryon we get

$$\sigma^{\mathrm{p}\,\vec{\mathrm{p}}\to\mathrm{p}\vec{\Lambda}^0\mathrm{K^+}}(\varepsilon) = \sigma^{\mathrm{pp}\to\mathrm{p}\Lambda^0\mathrm{K^+}}(\varepsilon) \times\Big(1+\frac{1}{3}\,(\vec{n}\cdot\vec{\zeta})(\vec{n}\cdot\vec{\zeta}_{\Lambda^0})\Big),$$
$$\sigma^{\mathrm{p}\,\vec{\mathrm{p}}\to\mathrm{p}\Sigma^0\mathrm{K^+}}(\varepsilon) = \sigma^{\mathrm{pp}\to\mathrm{p}\Sigma^0\mathrm{K^+}}(\varepsilon) \times\Big(1+\frac{1}{3}\,(\vec{n}\cdot\vec{\zeta})(\vec{n}\cdot\vec{\zeta}_{\Sigma^0})\Big). \quad (4.6)$$

The cross-sections for unpolarized baryons $\sigma^{\mathrm{pp}\to\mathrm{p}\Lambda^0\mathrm{K^+}}(\varepsilon)$ and $\sigma^{\mathrm{pp}\to\mathrm{p}\Sigma^0\mathrm{K^+}}(\varepsilon)$ have been tabulated in ref. [1] for excess of energy $\varepsilon$ ranging values from the region $0.68\,\mathrm{MeV}\le\varepsilon\le 138\,\mathrm{MeV}$. Theoretical cross-sections fit experimental data with accuracy better than 11%.

## 5 Conclusion

We have shown that in our approach [1] to the description of pp reactions $p + p \to p + \Lambda^0 + K^+$ and $p + p \to p + \Sigma^0 + K^+$ near thresholds of the final states $p\Lambda K^+$ and $p\Sigma^0 K^+$, based on pp rescattering in the initial state with a dominant contribution of the one-pion exchange and strong final-state interaction of daughter hadrons, polarization properties of strange baryons can be investigated with respect to polarizations of colliding protons. We would like to accentuate that for the derivation of the effective Lagrangian (1.1) and the calculation of the amplitude of pp rescattering in the initial state we have used renormalizable *pseudoscalar* meson-baryon-baryon couplings that always fit data well [13].

Near thresholds of the reactions $p + p \to p + \Lambda^0 + K^+$ and $p + p \to p + \Sigma^0 + K^+$ we predict production of $p\Lambda^0$ and $p\Sigma^0$ pairs only in the spin singlet $^1S_0$ and spin triplet $^3S_1$ states. This result has been obtained by means of relativistically covariant partial-wave analysis worked out by Anisovich *et al.* for nucleon-nucleon scattering [3]. In order to implement this analysis to reactions $p + p \to p + \Lambda^0 + K^+$ and $p + p \to p + \Sigma^0 + K^+$ for the description of wave functions of $p\Lambda^0$ and $p\Sigma^0$ pairs we have generalized the projection operators introduced by Anisovich *et al.* for nucleon-nucleon pairs onto the case of baryon-baryon pairs with non-equal masses of coupled baryons.

In our model production of a polarized strange baryon can come about only for $p\Lambda^0$ and $p\Sigma^0$ pairs produced in the spin triplet state $^3S_1$. The more detailed predictions for polarization of strange baryons can be obtained from the theoretical cross-sections (4.5) and (4.6) in accord with specific experimental conditions of the experimental analysis of the reactions $p + p \to p + \Lambda^0 + K^+$ and $p + p \to p + \Sigma^0 + K^+$.

Now let us compare our results with the model-independent analysis of polarization of strange baryons in the reaction $p + p \to p + Y + K^+$ worked out by Rekalo *et al.* [11]. According to ref. [11] the most general form of the cross-section for the reaction $\vec{p} + \vec{p} \to p + Y + K^+$ with polarized colliding protons and unpolarized strange baryon should read

$$\sigma^{\vec{p}\,\vec{p}\to pYK^+}(\varepsilon) = \sigma^{pp\to pYK^+}(\varepsilon) \times \Big(1 + \mathcal{A}_1\,\vec{\zeta}_1\cdot\vec{\zeta}_2 + \mathcal{A}_2\,(\vec{n}\cdot\vec{\zeta}_1)(\vec{n}\cdot\vec{\zeta}_2)\Big), \quad (5.1)$$

where $\mathcal{A}_i\,(i = 1, 2)$ are real functions obeying the constraint

$$3\mathcal{A}_1 + \mathcal{A}_2 = 1. \quad (5.2)$$

When matching the expression (5.1) with ours (4.5) we find that

$$\mathcal{A}_1 = \frac{1}{3}\,, \qquad \mathcal{A}_2 = 0. \quad (5.3)$$

This agrees completely with Rekalo's prediction (5.2).

Unlike our results Rekalo *et al.* did not give an explicit expression of the cross-section for the reaction $p + p \to p + Y + K^+$ with polarized colliding protons and strange baryon. Therefore, we cannot compare our theoretical cross-sections (4.5) and (4.6) with analogous expressions which could be obtained within a model-independent approach [11].

However, following general properties of strong interactions and parity invariance, in particular, Rekalo *et al.* have predicted a dynamical polarization vector $\vec{P}_Y$ of a strange baryon in terms of a polarization vector of one of the colliding protons $\vec{\zeta}$ (see eq. (10) of ref. [11]). In our case the dynamical polarization vector $\vec{P}_Y$ is defined by

$$\vec{P}_Y = -\frac{2}{3}\,\vec{\zeta} + \frac{4}{3}\,\vec{n}\,(\vec{n}\cdot\vec{\zeta}). \quad (5.4)$$

This result can be verified experimentally[1].

The absence in our cross-sections (4.5) of the terms $\vec{\zeta}_Y\cdot(\vec{\zeta}_1\times\vec{\zeta}_2)$, $(\vec{n}\cdot\vec{\zeta}_Y)(\vec{n}\cdot(\vec{\zeta}_1\times\vec{\zeta}_2))$ and so testifies that in our approach polarization observables of strange baryons defining the cross-section for the reaction $p + p \to p + Y + K^+$ are even under time reversal, $T$-even polarization observables [11]. According to Rekalo's model-independent analysis this assumes the relation between phases of amplitudes of pp and pY scattering

$$\delta^{pYK^+}_{pp}(^3P_0; |\vec{p}\,|) + \delta_{pY}(^1S_0; q_{pY}) = \delta^{pYK^+}_{pp}(^3P_1; |\vec{p}\,|) + \delta_{pY}(^3S_1; q_{pY}), \quad (5.5)$$

where $\delta^{pYK^+}_{pp}(^3P_0; |\vec{p}\,|)$ and $\delta^{pYK^+}_{pp}(^3P_1; |\vec{p}\,|)$ are the phases of amplitudes of strong pp rescattering in the $^3P_0$ and $^3P_1$ states, respectively, and $\delta_{pY}(^1S_0; q_{pY})$ and $\delta_{pY}(^3S_1; q_{pY})$ are the phases of low-energy elastic pY scattering in the spin singlet $^1S_0$ and spin triplet $^3S_1$ states, respectively. Since scattering lengths and effective ranges of elastic pY scattering have been set equal this implies that $\delta_{pY}(^1S_0; q_{pY}) = \delta_{pY}(^3S_1; q_{pY})$. Substituting this relation into (5.5), we obtain the constraint

$$\delta^{pYK^+}_{pp}(^3P_0; |\vec{p}\,|) = \delta^{pYK^+}_{pp}(^3P_1; |\vec{p}\,|). \quad (5.6)$$

Hence, any experimental measurement of the cross-section for the reaction $p + p \to p + Y + K^+$ with non-vanishing contributions of $T$-odd polarization observables like $\vec{\zeta}_Y\cdot(\vec{\zeta}_1\times\vec{\zeta}_2)$ should evidence a violation of constraints (5.5) and (5.6). The former might mean that either scattering lengths and effective ranges of low-energy elastic pY scattering are not really equal for the spin singlet $^1S_0$ and spin triplet $^3S_1$ states or, in reality, amplitudes of strong pp rescattering in the $^3P_0$ and $^3P_1$ states of colliding protons differ themselves near threshold of the reaction $p + p \to p + Y + K^+$.

We are grateful to Prof. M.P. Rekalo for helpful discussions.

[1] The dynamical polarization vector $\vec{P}_Y$ can be related to the polarization vector $\vec{\zeta}_Y$ as follows $\vec{P}_Y = w_{pp\to pYK^+}\,\vec{\zeta}_Y$, where $w_{pp\to pYK^+}$ is a probability of the production of a strange baryon Y in the reaction $p + p \to p + Y + K^+$.

## References


1. A.Ya. Berdnikov, Ya.A. Berdnikov, A.N. Ivanov, V.F. Kosmach, M.D. Scadron, N.I. Troitskaya, Eur. Phys. J. A **9**, 425 (2000), hep-ph/0011343.
2. M.M. Nagels *et al.*, Nucl. Phys. B **147**, 253 (1979).
3. V.V. Anisovich, M.N. Kobrinsky, D.I. Melikhov, A.V. Sarantsev, Nucl. Phys. A **544**, 747 (1992); V.V. Anisovich, D.I. Melikhov, B.Ch. Metch, H.R. Petry, Nucl. Phys. A **563**, 549 (1993).
4. J.T. Balewski *et al.*, Phys. Lett. B **388**, 859 (1996).
5. J.T. Balewski *et al.*, Phys. Lett. B **420**, 211 (1998).
6. R. Bilger *et al.*, Phys. Lett. B **420**, 217 (1998).
7. A.N. Ivanov, V.A. Ivanova, H. Oberhummer, N.I. Troitskaya, M. Faber, Eur. Phys. J. A **12**, 87 (2001), nucl-th/0108067.
8. A.N. Ivanov, H. Oberhummer, N.I. Troitskaya, M. Faber, Eur. Phys. J. A **7**, 519 (2000), nucl-th/0006049; **8** (2000) 129, nucl-th/0006050; 223 (2000), nucl-th/0006051.
9. J.T. Balewski *et al.*, Eur. Phys. J. A **2**, 99 (1998).
10. COSY-11 Collaboration (S. Sewerin *et al.*), Nucl. Phys. A **663**, 473 (2000); Phys. Rev. Lett. **83**, 682 (1999).
11. M.P. Rekalo, J. Arvieux, E. Tomasi-Gustafsson, Phys. Rev. C **56**, 2238 (1997).
12. Ya.A. Berdnikov, A.N. Ivanov, V.F. Kosmach, N.I. Troitskaya, Phys. Rev. C **60** 015201 (1999); A.Ya. Berdnikov, Ya.A. Berdnikov, A.N. Ivanov, V.F. Kosmach, M.D. Scadron, N.I. Troitskaya, Phys. Rev. D **64**, 014027 (2001).
13. L.R. Thebaud, M.D. Scadron, Phys. Rev. D **8**, 2190 (1973); S.A. Coon, M.D. Scadron, B.R. Barrett, Nucl. Phys. A **249**, 483 (1975); H.F. Jones, M.D. Scadron, Phys. Rev. D **11**, 174 (1975); G.E. Hite, R.J. Jacob, M.D. Scadron, Phys. Rev. D **14**, 1306 (1976); B.H. Wilde, S.A. Coon, M.D. Scadron, Phys. Rev. D **18**, 4489 (1978); S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt, B.H.J. McKellar, Nucl. Phys. A **317**, 242 (1979); S.A. Coon, M.D. Scadron, Phys. Rev. C **23**, 1150 (1981); **26**, 2402 (1982); M.D. Scadron, Ann. Phys. (N.Y.) **159**, 184 (1985); S.A. Coon, M.D. Scadron, Phys. Rev. C **42**, 2256 (1990); R.E. Karlsen, W.H. Ryan, M.D. Scadron, Phys. Rev. D **43**, 157 (1991); R.E. Karlsen, M. D, Scadron, Phys. Rev. D **43**, 1739 (1991); Z. Phys. C **52**, 325 (1991); Phys. Rev. D **45**, 4113 (1992); M.D. Scadron, Phys. Rev. D **57**, 5307 (1998); A. Bramon, R. Escribano, M.D. Scadron, Eur. Phys. J. C 7, 271 (1999); M.D. Scadron, Mod. Phys. Lett. A **14**, 1349 (1999); M.D. Scadron, D. Tadic, J. Phys. G **27**, 163 (2001), hep-ph/0011328.